SHORT PAPER IN MATHEMATICAL FINANCE

Correlation versus Cointegration: Do Cointegration based Index-Tracking Portfolios perform better?

Evidence from the Swedish Stock-Market

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Abstract

Passive portfolio management which aims to replicate a stock index faces basically two different optimization methods. Traditional portfolio management employs historical stock return data of preselected stocks in order to replicate the underlying stock index. The cointegration method employs time series data of stock prices instead, even though stock price data may statistically often exhibit random walk behavior. In this review the advantage of the latter method could be asserted. Thereby, different stock portfolios with respect to the Swedish stock market are constructed which rest upon both, the concept of correlation and the concept of cointegration. The cointegration based models dominate, which can be ascertained by comparing their Sharpe ratios as well as their Treynor ratios. The dominating stock portfolio beat the index by 79.08% within the overall 10-years out-of-sample period, whereas the annual volatility on average was 1.10 base points lower.

Keywords: Cointegration models, Index tracking, Quasi-maximum-likelihood estimation, Correlation models

Korrelation kontra Kointegration: Leisten kointegrationsbasierte Index-Tracking-Portfolios mehr?

Evidenz vom schwedischen Aktienmarkt

Zusammenfassung

Passives Portfolio-Management, das darauf abzielt, einen Aktienindex zu replizieren, hat grundsätzlich zwei verschiedene Optimierungsmethoden zur Verfügung. Traditionelles Portfolio-

Management benutzt historische Zeitreihen, um mit Renditenzeitreihen eines zuvor selektierten Aktienpools die zugrunde liegenden Indexrenditen zu replizieren. Die Kointegrationsmethode verwendet dagegen die Zeitreihen der Aktienpreise, die sich statistisch betrachtet wie Random Walks verhalten. In diesem Beitrag konnte der Vorteil der letzteren Methode herausgestellt werden. Hierbei wurden für den schwedischen Aktienmarkt verschiedene Portfolios konstruiert, die sowohl auf dem Korrelationskonzept als auch auf dem Kointegrationskonzept beruhen. Die Kointegrationsmodelle dominieren hierbei, was sich anhand der Vergleiche der Sharpe-Ratios wie auch der Treynor-Ratios zeigen lässt. Das dominierende Portfolio hat in dem betrachteten 10-Jahreszeitraum den Index um insgesamt 79.08% geschlagen, wobei die jährliche Volatilität durchschnittlich um 1.10 Basispunkte geringer war.

Schlüsselwörter: Kointegrationsmodelle, Index-Tracking, Quasi-Maximum-Likelihood Schätzung, Korrelationsmodelle

Introduction

Passive index tracking strategies aim to exhibit the same expected return like the underlying stock market while involving a volatility being as low as possible. In contrast to traditional portfolio optimization models employing stock returns, as suggested by Roll (1992), Alexander (1999) introduces optimization models being based on cointegration analysis. Alexander and Dimitriu (2005a) compare in their empirical analysis concerning the S&P 500® stock market these two approaches and find out that no significant advantages or limitations of a cointegration relationship with the benchmark are empirically evident irrespective if weight constraints are taken into account or not.[1] But is empirically data of other countries' stock markets supporting this outcome? Since cointegration exhibits low volatility even under volatile market circumstances, it may be advantageously to employ cointegration optimal portfolios especially in down market movements. In the following work, an empirical long-run analysis gives evidence that cointegration optimal index-tracking models perform significantly better compared to their traditional counterparts. Thereby, the Swedish stock market is the focus of this study. The Swedish leading stock index OMS measures the performances of 30 companies in the Swedish stock market which exhibit the highest market capitalization.

Background

In line with the seminal work of Markowitz (1959), Sharpe (1964) and Black (1972), the traditional statistical tool for portfolio optimization is correlation analysis of asset returns focusing on minimizing the variance of a tracking error. Alexander (1999) argues that correlation is intrinsically a short run measure and the tracking error of stock portfolios being based on correlation analysis can exhibit out of sample random walk behavior.

Cointegration, as defined and developed by Granger (1981) and Engle and Granger (1987), is a property of some nonstationary time series. If two or more nonstationary time series are cointegrated, a linear combination relationship being stationary is said to exist. In the context of portfolio allocation, whether the value series of a fixed weight portfolio of assets with nonstationary prices is stationary, the assets will exhibit a cointegrated set. The set of asset weights generating such a portfolio is called the cointegrating vector.

[1] The S&P 500® measures the performance of 500 companies in the US-stock market which exhibit the highest market capitalization.

In contrast to correlation based models, Alexander and Dimitriu (2005a) show that the tracking error of cointegration optimal portfolios exhibits stationarity even out of sample, as cointegration ensures the reversion of the asset to the underlying benchmark.

Statistical Model

In line with Roll (1992) the optimization problem is given by minimizing the tracking error variance of the following model:

$$r_{OMX,t} = a_1 r_{1,t} + ... a_N r_{N,t} + \varepsilon_t \quad (1)$$

where $r_{OMX,t}$ denotes the log index returns and $r_{i,t}$ denotes the log returns of stocks $i = 1,...,N$. The optimization method being employed here is Quasi-Maximum Likelihood-Estimation (QMLE). Hence, the mean-variance optimal portfolio may be estimated by maximizing the log-likelihood being given by

$$\log L(\theta,t) = -\frac{T}{2}\log(2\cdot\pi) - \frac{T}{2}\log\sigma^2 - \frac{1}{2}\sum_{t\in T}\left(\frac{\varepsilon_t^2}{\sigma^2}\right), \quad (2)$$

where $\varepsilon_t = r_{OMX,t} - \sum_{i=1}^{N} a_i r_{i,t}$.

In line with van Montefort, Visser and and Fijn van Draat (2008) it is usual to impose weight restrictions. In the following analysis, five restrictions are imposed. First, the number stocks being employed are preselected and for practical purposes not changing over time.

Second, the stock weights sum up to one being given by Equation (3). The third usual restriction is that no short sales are allowed being given by Equation (4) and the fourth restriction which should be accounted for is that the annual turnover is less than c_1 of the overall portfolio volume which is given by Equation (5). The last restriction being imposed is that the asset weights are equal or less than c_2 in order to avoid being overinvested in one stock (see Equation 6).

$$\sum_{i=1}^{N} a_i = 1 \quad (3)$$

$$a_i > 0 \quad \text{for} \quad i = 1,...,N \quad (4)$$

$$\sum_{i=1}^{N} \left| a_{k+1,i} - a_{k,i} \right| \le c_1 \quad (5)$$

$$a_i < c_2 \quad \text{for} \quad i = 1,...,N \quad (6)$$

The index k of Equation (5) denotes the rebalancing moments $k = 1,...,K$. These restrictions are valid for both models, the models being based on correlation analysis and the models being based on cointegration analysis.

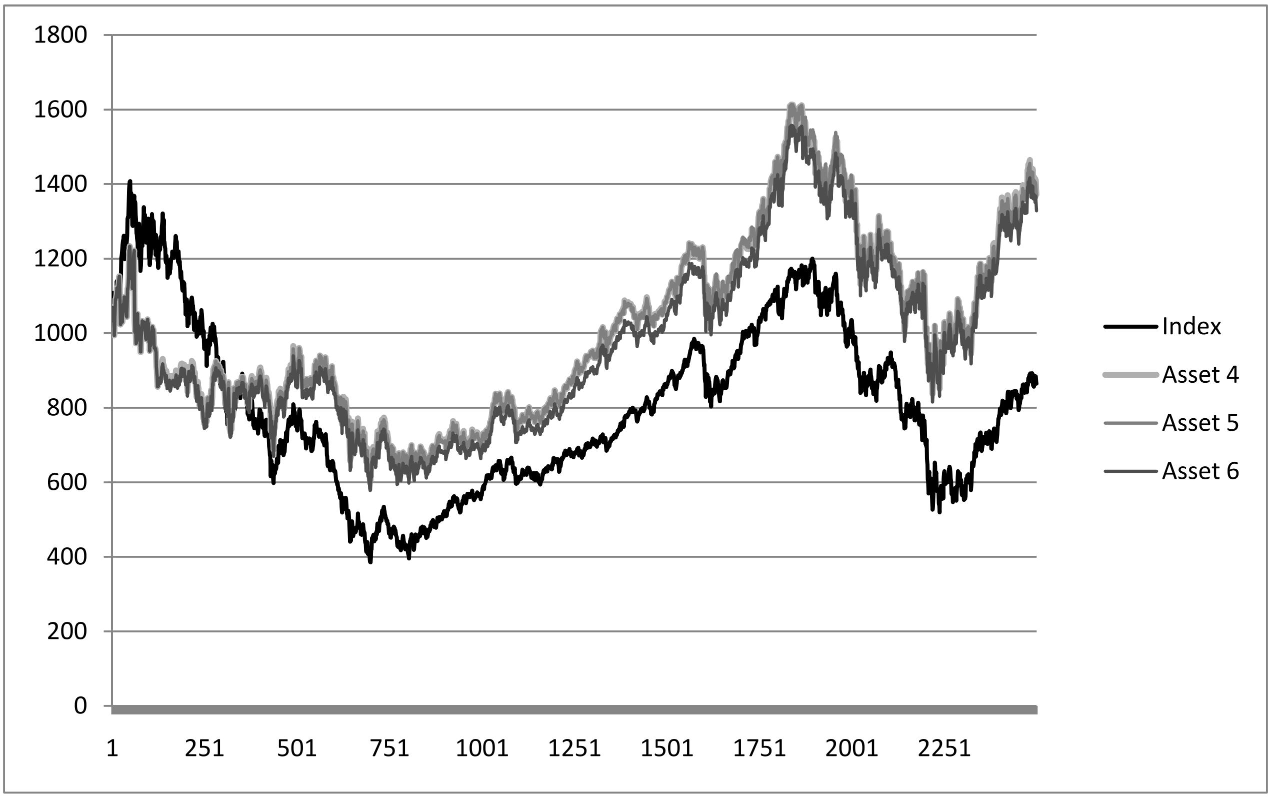
Constructing cointegration optimal portfolios, however, involves in accordance to Alexander and Dimitriu (2005a) first of all running the optimization procedure (see Equation 2) by employing the logarithm of the stock prices $p_{i,t}$ instead of the log-returns $r_{i,t}$ (see Equation 1). The second additional step is testing for cointegration. Alexander and Dimitriu (2005a) suggest employing the augmented Dickey fuller test (ADF) given by

$$\Delta\hat{\varepsilon}_t = \gamma\hat{\varepsilon}_{t-1} + \sum_{l=1}^{L} \alpha_l \Delta\hat{\varepsilon}_{t-l} + u_t . \quad (7)$$

Thereby, the null hypothesis tested is of no cointegration, i.e. $\gamma = 0$, against the alternative of $\gamma < 0$.[2] Whether the null hypothesis of no cointegration is rejected, the cointegration-optimal tracking portfolio based on QMLE of Equation (2) is expected to have very similar returns to the market index. Afterwards the overall out-of-sample period concerning all constructed portfolios is tested for cointegration by employing the trace-test as suggested by Johansen (1988).

[2]The critical values for the t-statistic of y are obtained using the response surfaces provided by MacKinnon (1991).

Exhibit 1: The cointegration optimal portfolios and the OMX 10 years out-of-sample



Note: The out-of-sample period runs from 31.12.1999-31.12.2009 and includes 2514 observations. The Overall return of the OMX was -2.06% p.a., whereas the index volatility was 27.23% p.a.

Limitation of the data set

The stock market data is available for free on the index provider's homepage.[3] Operating with long-run data involves that researchers may face a survivorship bias. That means that there are only certain stocks available for which daily data is available until 31.12.1996. The out-of-sample period runs from 31.12.1999-31.12.2009 and contains 2514 observations. The stocks being employed are shown in panel 1. Even though the number of stock that can be employed is limited, all relevant business sectors can be accounted for.

Results

Due to survivorship bias 17 of 30 stocks (i.e. 56.67%) could be taken into account corresponding to 57.20% of the overall market capitalization (i.e. on the 4. Jan 2010).

Panel 1: Preselected stocks of the OMX due to data limitations

Stock	Sector	Market capitalization	Symbol
Assa Abloy	Industry	1.73 %	ASSA B
Atlas Copco A	Industry	3.66%	ATCO A
Atlas Copco B	Industry	1.55%	ATCO B
Electrolux	Electronic	2.08%	ELUX
Ericsson	Telecom	8.22%	ERIC B
Getinge	Healthcare	1.26%	Getinge
Hennes & Mauritz	Fashion industry	12.02%	HM B
Investor	Financial Services	2.51%	INVE B
SCA	Healthcare	2.38%	SCA B
SEB	Banks	3.98%	SEB A
Securitas	Security Services	1.01%	SECU B
Svenska Handelsbanken	Banks	5.17%	SHB A
Skanska	Construction	2.01%	SKA B
SKF	Industry	2.01%	SKF B
SSAB	Commodity/Steel	1.22%	SSAB A
Swedbank	Banks	2.76%	SWED A
Volvo Group	Automobile	3.63%	VOLV B

[3] http://www.nasdaqomxnordic.com/indexes

Three different rebalancing strategies are considered for both methods correlation and cointegration.

Considering annual rebalancing, the number of rebalancing moments is $K = 10$, whereas $K = 20$ concerning the half-yearly rebalancing strategy. In the following empirical analysis, the parameter c_1 and c_2 are set equal to 10%, which means that the overall portfolio turnover from one allocation moment to the next one may not be larger than 10% of the overall portfolio volume. Moreover, it is not allowed to allocate a weight being larger than 10% to only one stock. In line with Alexander and Dimitriu (2005a) three years of daily data is used to estimate the weights of the models. The first calibration period runs from 31.12.1996-31.12.1999. Depending on the rebalancing strategy, the time window being employed for estimate the stock weights is moved forward one year or six months only.

Considering the Treynor ratio, only, Panel 2 shows that cointegration optimal portfolios dominate their traditional counterparts. Even if taking into account both, the Sharp ratio and the Treynor ratio as well, the half-yearly and annual rebalanced cointegration optimal portfolios perform the best. However, the overall out-of-sample data shows that neither the cointegration optimal (i.e. in-sample cointegration optimal) portfolio nor the correlation based models exhibit out-of-sample a cointegration relationship with the benchmark, as the p-values of the corresponding trace-tests are smaller than 0.05.

Panel 2: Index-tracking strategies and their statistical properties

	Asset 1	Asset 2	Asset 3	Asset 4	Asset 5	Asset 6
Method	Correlation-Analysis			Cointegration-Analysis		
Rebalancing moments	Half-yearly	Annual	Buy-and-Hold	Half-yearly	Annual	Buy-and-Hold
Total turnover	41.04%	32.19%	-	21.99%	20.59%	-
Annual turnover	4.00%	3.20%	-	2.20%	2.06%	-
Total gross return	48.01%	50.95%	55.10%	59.12%	58.46%	55.65%
Annual gross returns	4.81%	5.10%	5.51%	5.91%	5.85%	5.56%
Total trading costs	4.10%	3.22%	-	2.20%	2.06%	-
Annual trading costs	0.04%	0.03%	-	0.02%	0.02%	-
Annual net returns	4.77%	5.07%	5.51%	5.89%	5.83%	5.56%
Annual volatility	25.49%	25.46%	25.27%	26.13%	26.12%	26.04%
Sharpe Ratio	0.19	0.20	0.22	0.23	0.22	0.21
Rank (Sharpe Ratio)	6	5	3	1	2	4
Treynor Ratio	5.98	6.37	6.94	7.64	7.56	7.20
Rank (Treynor Ratio)	6	5	4	1	2	3
p-value (Trace-Test)	0.94	0.94	0.93	0.11	0.12	0.11
Annual standard deviation $\hat{\varepsilon}_t$	13.31%	13.30%	13.04%	15.52%	15.48%	15.30%

Discussion

In contrast to Alexander and Dimitriu (2005a) some of their outcomes cannot be supported. The cointegration optimal tracking portfolios dominate significantly their traditional counterparts (i.e. assets 1-3 in panel 2). Even the “buy and hold” strategy would overperform better than all correlation based models if the Treynor ratio is considerd only. Unlike Alexander and Dimitriu (2005a), the Sharp ratios of the cointegration optimal portfolios are better than their correlation based counterparts apart from the “buy and hold” strategy. They report furthermore that the correlation based models generate marginally lower transaction costs. However, panel 1 shows that the correlation models’ trading costs for both rebalancing strategies are higher, the half-yearly strategy (i.e. 86.36% higher costs) and the annual one (56.31% higher costs).

However, it can be supported that cointegration based models exhibit higher tracking error volatility, as also mentioned by Alexander and Dimitriu (2005a). Apart from that exhibit 1 shows that especially when the market crashes, cointegration based models exhibit an advantegeous due to a price equilibrium being estimated from a long historical sample. Alexander and Dimitriu (2005b) argue that in bubble formations the normal equilibrium mechanism is not working, whereas cointegration optimal tracking portfolios overperform, as they are based on a long-run price equilibrium.

Concluding remarks

The benefits from cointegration optimal index tracking portfolios are clear. If a bubble is formed up in the stock market, the cointegration optimal portfolios exhibit defensive properties being a result of an implicit market timing factor. If the market switches upwards, cointegration ensures the tracking portfolios to be tied closely to the underlying benchmark. In the Swedish stock market, the cointegration optimal portfolios performed at least 7.63% p.a. better than the stock index whereas the volatility was at least 1.19 base points lower.[4]

[4] The “buy and hold” strategy exhibited 5.56% annual net returns and 26.04% annual volatility, whereas the OMX stock index exhibited -2.06% annual returns and 27.23% annual volatility concerning the out-of-sample period running from 31.12.1999-31.12.2009.

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